HW 4

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E 4.1

$P(x) : x > 1$,
$Q(x) : x^2 < 16$
$R(x) :$ $x + 1$ is even.

a) $S = \{2, 3\}$
b) $S = \{-1, -3, 1, 3\}$
c) $S = \{x \in Z | x \leq 2, x = 2k$ for some k in $\mathbb{Z}\}$

Q	P	R	$\neg P$	$(Q \wedge \neg P)$	¬R	$(Q \wedge \neg P) \wedge \neg R$
T	T	T	F	F	F	F
T	T	F	F	F	T	T
T	F	T	T	T	F	F
T	F	F	T	T	T	F
F	T	T	F	F	F	F
F	T	F	F	F	T	T
F	F	T	T	F	F	F
F	F	F	T	F	T	T

d) Logical equivalence:

Beginning statement:
$(P(x) \Rightarrow Q(x)) \Rightarrow R(x)$

A => B is equivalent to ¬A ∨ B
$\neg(P(x) \Rightarrow Q(x)) \vee R(x)$
$\neg(\neg P(x) \vee Q(x)) \vee R(x)$

De Morgan's Laws
$(\neg\neg P(x) \wedge \neg Q(x)) \vee R(x)$
$(P(x) \wedge \neg Q(x)) \vee R(x)$

Therefore:
$S = \{x \in Z | x > 4$ or x is odd$\}$

E 4.2

The statement "If x is an odd integer, then $\frac{3x+5}{2}$ is an odd integer" is logically equivalent to "x is an even integer or $\frac{3x+5}{2}$ is an odd integer"

1: false - 1 is not an even integer nor is $\frac{3(1)+5}{2} = 4$ an odd integer.
2: true - 2 is an even integer.
3: true - $\frac{3(3)+5}{2} = 7$ is an odd integer
4: true - 4 is an even integer
5: false - 5 is not an even integer nor is $\frac{3(5)+5}{2} = 10$ an odd integer.
6: true - 6 is an even integer

The open statement $\forall x \in Z, P(x)$ is false since there are some values (for example 1 & 5) for which $P(x)$ is false.

E 4.3

The statement Q(x): If x is an even integer, then $3x + 5$ is an odd integer is logically equivalent to "x is an odd integer or $3x + 5$ is an odd integer"

1: true - 1 is an odd integer.
2: true - $3(2) + 5 = 11$ is an odd integer
3: true - 3 is an odd integer
4: true - $3(4) + 5 = 17$ is an odd integer
5: true - 5 is an odd integer
6: true - $6(4) + 5 = 29$ is an odd integer

The statement $\forall x \in Z, Q(x)$ appears to be true based on the values tested.

E 4.4**

$\forall x \in U, (x \in A) \Leftrightarrow (x \in B)$

E 4.5

a) $\exists x \in \mathbb{Z}, 4 < x < 6$

b) $\forall x \in \mathbb{Z}, x$ is odd $\implies x^2$ is odd

c) $\forall x \in \mathbb{Z}, x^2 \in ODD \implies x \in ODD$

d $\forall x \in \mathbb{R}, x \notin \mathbb{Q} \implies x \neq 0$

e) $\forall x \in \mathbb{r}, \forall y \in \mathbb{R}, x \in \mathbb{Q} \land y \in \mathbb{Q} \implies (x + y) \in \mathbb{Q}$

f) $\forall x \in \mathbb{R}, x^2 \implies (x > 0 \land x \in \mathbb{R})$

g) $\exists x \in \mathbb{Z}, x^2 - 5x + 6 = 0$

h) $\forall x \in \mathbb{R}, x2 - 5x + 6 = 0 \implies x \in \mathbb{Z}$

E 4.6

a) There exists some value x in the real numbers such that $x^2 = 2$

b) For all values x that are integers, x is even if and only if x^2 is even

c) For all values x in the real numbers, if $x > 1$ then $x^3 > 1$.

d) For all values x in the real numbers, if $x^2 - 2x + 1 = 0$ then $x = 1$

e) There exists some value x in the rational numbers such that $2x^3 - x^2 + 2x - 1 = 0$

f) For every real number x, there exists an integer y and a number z in the range 0 (inclusive) to 1 (not inclusive) such that x is the sum of y and z.